# Estimations with Python

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## Contents



## 1 Introduction

This chapter aims to show how to compute point estimates and confidence intervals. We explore the following cases:

- Estimation and confidence interval of the mean,
- Comparing the confidence of the means by group
- Estimation and Confidence interval of the proportion
- Confidence interval of the difference of proportions.

We start by importing the data in the file `data1.csv`

[1]:
```
import pandas as pd
import numpy as np
df = pd.read_csv('data1.csv')
df.head(5)
```

[1]:
```
   Age Attrition     BusinessTravel  DailyRate              Department  \
0   41       Yes      Travel_Rarely       1102                   Sales
1   49        No  Travel_Frequently        279  Research & Development
2   37       Yes      Travel_Rarely       1373  Research & Development
3   33        No  Travel_Frequently       1392  Research & Development
4   27        No      Travel_Rarely        591  Research & Development

   DistanceFromHome  Education EducationField  EmployeeCount  EmployeeNumber  \
```

```
0                  1          2  Life Sciences              1               1
1                  8          1  Life Sciences              1               2
2                  2          2          Other              1               4
3                  3          4  Life Sciences              1               5
4                  2          1        Medical              1               7

   ...  RelationshipSatisfaction StandardHours  StockOptionLevel  \
0  ...                         1            80                 0
1  ...                         4            80                 1
2  ...                         2            80                 0
3  ...                         3            80                 0
4  ...                         4            80                 1

   TotalWorkingYears  TrainingTimesLastYear WorkLifeBalance  YearsAtCompany  \
0                  8                      0               1               6
1                 10                      3               3              10
2                  7                      3               3               0
3                  8                      3               3               8
4                  6                      3               3               2

  YearsInCurrentRole  YearsSinceLastPromotion  YearsWithCurrManager
0                  4                        0                     5
1                  7                        1                     7
2                  0                        0                     0
3                  7                        3                     0
4                  2                        2                     2

[5 rows x 35 columns]
```

# 2 The mean

## 2.1 The theory

Assume that we're interested in the variable age from the imported data, and we would like to know the following information:

- the average age of the employees in the survey?
- the probability that one given employee has an age higher than 50?
- the distribution of the employees' age between the different departments?

We will first assume that sequence of employees's age is a random sample. We will write as a sequence of random variables $X_1, \ldots, X_n$.

We assume also that $X_1, \ldots, X_n$ are generated from a Normal distribution with mean $\mu$ and with variance $\sigma^2$.

It's known that the $\overline{X}$ is an estimator of the mean $\mu$. Since the sample mean is also a random variable with normal distribution with mean $\mu$ and variance $\frac{\sigma^2}{n}$, we can provide an interval that provides the error on the estimation. It's called the **Confidence Interval**.

We aim in chapter to show how to compute the confidence interval of the mean $\mu$ with level $(1-\alpha)$, $\alpha \in (0,1)$. We denoted by $\mathrm{CI}_{1-\alpha}(\mu)$

If $\sigma^2$ is **known**, $\mathrm{CI}_{1-\alpha}(\mu)$ is expressed as follows:

$$\left(\overline{X} - z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}},\ \overline{X} + z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

where

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

is the sample mean and $z_{1-\alpha/2}$ is the percentile associated to $(1-\alpha/2)$ from the standard normal distribution:

$$F_Z(z_{1-\alpha/2}) = 1-\alpha/2$$

where $Z$ is a random variable with standard normal distribution.

If $\sigma^2$ is **unknown**, $\mathrm{CI}_{1-\alpha}(\mu)$ is expressed as follows:

$$\left(\overline{X} - t_{1-\alpha/2,n-1}\frac{S}{\sqrt{n}},\ \overline{X} + t_{1-\alpha/2,n-1}\frac{S}{\sqrt{n}}\right)$$

where $S^2$ is the sample mean:

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

and $t_{1-\alpha/2,n-1}$ is the percentile associated to $(1-\alpha/2)$ from the $t-$distribution with $n-1$ degrees of freedom:

$$F_{T_{n-1}}(t_{1-\alpha/2,n-1}) = 1-\alpha/2$$

where $T_{n-1}$ is a random variable with $t-$distribution $n-1$ degrees of freedom.

### 2.2 Practice with Python

[2]:
```
import numpy as np
from scipy.stats import norm,t
```

We will write two functions. A first one returns the $\mathrm{CI}_{1-\alpha}(\mu)$ when $\sigma^2$ is known and the second functions returns $\mathrm{CI}_{1-\alpha}(\mu)$ when $\sigma^2$ is unknown.

**1st function**

[3]:
```
def get_ci_known_variance(sigma, sample_mean, sample_size, alpha):
    margin_of_error = norm.ppf(1 - alpha/2)*sigma/np.sqrt(sample_size)
    return sample_mean - margin_of_error, sample_mean + margin_of_error
```

**2nd function**

[4]:
```python
def get_ci_unknown_variance(sample_s, sample_mean, sample_size, alpha):
    margin_of_error = t.ppf(1 - alpha/2,sample_size-1)*sample_s/np.
 ↪sqrt(sample_size)
    return sample_mean - margin_of_error, sample_mean + margin_of_error
```

**Practice**

simulating data

[5]:
```python
mu, sigma = 40, 2.5
random_sample = np.random.normal(mu, sigma, 100)
```

[6]:
```python
random_sample[0:5]
```

[6]:
```
array([38.03299039, 41.66455159, 41.73117252, 37.66321625, 37.82044562])
```

Computing the sample mean

[7]:
```python
sample_mean=np.average(random_sample)
```

[8]:
```python
sample_mean
```

[8]:
```
40.090871779180944
```

[9]:
```python
sample_size=len(random_sample)
sample_size
```

[9]:
```
100
```

$\text{CI}_{1-\alpha}(\mu)$ with known variance and $\alpha = 0.05$

[10]:
```python
get_ci_known_variance(sigma,sample_mean,sample_size,.05)
```

[10]:
```
(39.60088078304593, 40.58086277531596)
```

Computing the sample standard deviation

[11]:
```python
sample_s=random_sample.std()
sample_s
```

[11]:
```
2.7015723352282577
```

$\text{CI}_{1-\alpha}(\mu)$ with unknown variance and $\alpha = 0.05$

[12]:
```python
get_ci_unknown_variance(sample_s,sample_mean,sample_size,.05)
```

[12]:
```
(39.55482121685226, 40.626922341509626)
```

We can also use a function already implemented in the library `scipy` to compute $\text{CI}_{1-\alpha}(\mu)$ when the variance is unknown.

[13]:
```
confidence_level = 0.95
degrees_freedom = sample_size - 1
```

[14]:
```
import scipy
```

[15]:
```
sample_standard_error = scipy.stats.sem(random_sample)
```

[16]:
```
sample_standard_error
```

[16]:
```
0.2715182357562536
```

[17]:
```
sample_s/np.sqrt(sample_size-1)
```

[17]:
```
0.27151823575625356
```

[18]:
```
scipy.stats.t.interval(confidence_level, degrees_freedom, sample_mean,␣
 ↪sample_standard_error)
```

[18]:
```
(39.552120693149654, 40.629622865212234)
```

We can also use `scipy.stats.norm.interval` to compute $\mathrm{CI}_{1-\alpha}(\mu)$ with known variance

[19]:
```
scipy.stats.norm.interval(.95,loc=sample_mean, scale=sigma/np.sqrt(sample_size))
```

[19]:
```
(39.60088078304593, 40.58086277531596)
```

### 2.3 Practice with data

Application: comparing the average of the Daily rate between Men and Women.

[20]:
```
df['Gender'].value_counts()
```

[20]:
```
Male      882
Female    588
Name: Gender, dtype: int64
```

[21]:
```
df.groupby("Gender").agg({"DailyRate": [np.mean, np.std, np.size]})
```

[21]:
```
         DailyRate
              mean         std size
Gender
Female  808.273810  408.241680  588
Male    798.626984  400.509021  882
```

[22]:
```
x=df.groupby("Gender").agg({"DailyRate": [np.mean, np.std, np.size]})
```

[23]:
```
x=np.array(x)
```

[24]:
```
x
```

[24]:
```
array([[808.27380952, 408.24167967, 588.        ],
       [798.62698413, 400.50902101, 882.        ]])
```

The female sample size

[25]:
```
x[0,2]
```

[25]:
```
588.0
```

The female sample mean

[26]:
```
x[0,0]
```

[26]:
```
808.2738095238095
```

The female standard error

[27]:
```
x[0,1]/np.sqrt(x[0,2]-1)
```

[27]:
```
16.84993739332325
```

The `DailyRate` Female CI(95%)

[28]:
```
CI_F=scipy.stats.t.interval(.95, x[0,2] -1, x[0,0], x[0,1]/np.sqrt(x[0,2]-1))
```

[29]:
```
CI_F
```

[29]:
```
(775.1803043941346, 841.3673146534844)
```

We compute then the The DailyRate Female CI(95%)

We select now the `DailyRate`sample for Men and Women separatly

[30]:
```
CI_M=scipy.stats.t.interval(.95, x[1,2] -1, x[1,0], x[1,1]/np.sqrt(x[1,2]-1))
```

[31]:
```
CI_M
```

[31]:
```
(772.1438431601089, 825.1101250938593)
```

[32]:
```
CI_M[0]
```

[32]:
```
772.1438431601089
```

We can then visualize these Confidence intervals together to see the difference between the `DailyRate` means.

[33]:
```
ci_dailrate = {}
ci_dailrate['Gender'] = ['Male','Female']
ci_dailrate['lb'] = [CI_M[0],CI_F[0]]
ci_dailrate['ub'] = [CI_M[1],CI_F[1]]
df_ci= pd.DataFrame(ci_dailrate)
```

[34]:
```
df_ci
```

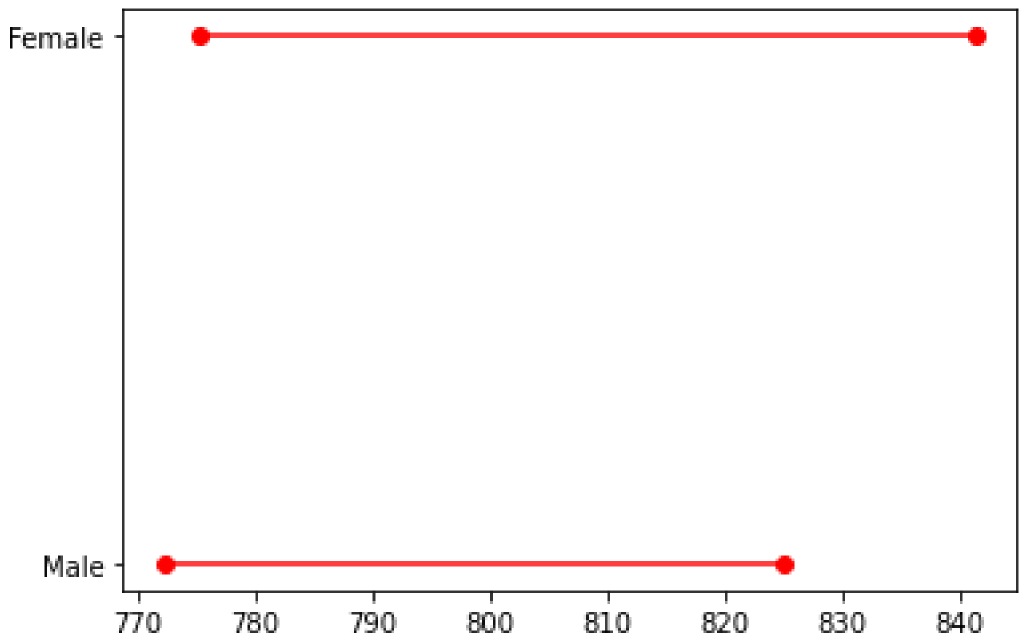
[34]:

| | Gender | lb | ub |
|---|---|---|---|
| 0 | Male | 772.143843 | 825.110125 |
| 1 | Female | 775.180304 | 841.367315 |

[35]:
```
import matplotlib.pyplot as plt
```

[36]:
```
for lb,ub,y in zip(df_ci['lb'],df_ci['ub'],range(len(df_ci))):
    plt.plot((lb,ub),(y,y),'ro-')
plt.yticks(range(len(df_ci)),list(df_ci['Gender']))
```

[36]:
```
([<matplotlib.axis.YTick at 0x1523a227130>,
  <matplotlib.axis.YTick at 0x1523a216970>],
 [Text(0, 0, 'Male'), Text(0, 1, 'Female')])
```



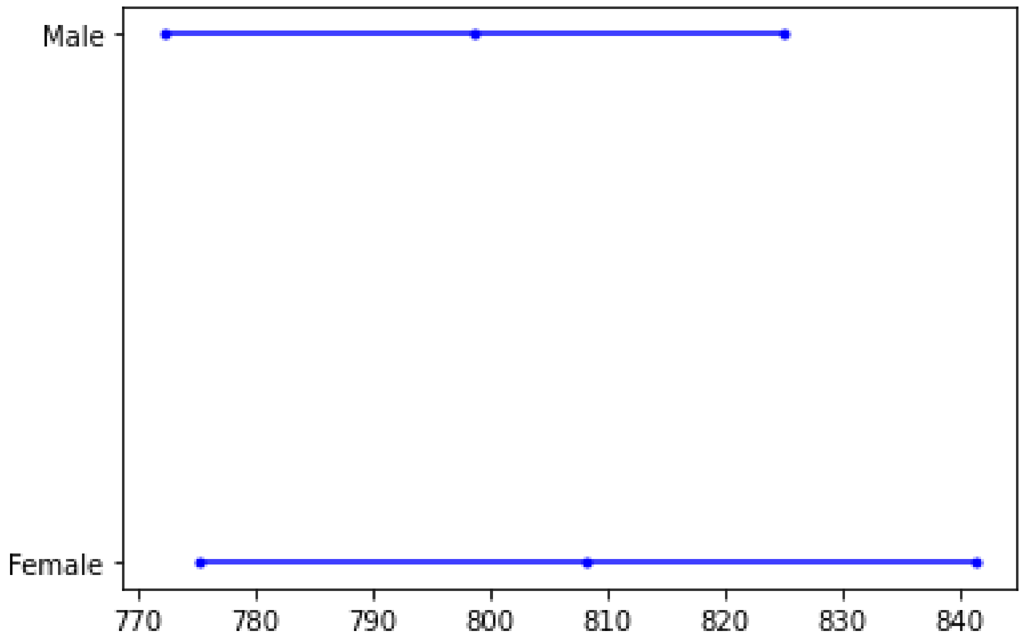

We will now write a Python function that can compare the Confidence Intervals of the means for a given continuous variable according to groups defined by a categorical variable.

[37]:
```
import pandas as pd
import numpy as np
import scipy.stats as st

def plot_diff_in_means(data: pd.DataFrame,alpha, col1: str, col2: str):
    """
    given data, plots difference in means with confidence intervals across groups
```

```python
    col1: categorical data with groups
    col2: continuous data for the means
    alpha: is the level of significance, it's usualy equal to .95
    """
    n = data.groupby(col1)[col2].count()
    # n contains a pd.Series with sample size for each category

    cat = list(data.groupby(col1, as_index=False)[col2].count()[col1])
    # cat has names of the categories, like 'category 1', 'category 2'

    mean = data.groupby(col1)[col2].agg('mean')
    # the average value of col2 across the categories

    std = data.groupby(col1)[col2].agg(np.std)
    se = std / np.sqrt(n)
    # standard deviation and standard error

    lower = st.t.interval(alpha = alpha, df=n-1, loc = mean, scale = se)[0]
    upper = st.t.interval(alpha = alpha, df =n-1, loc = mean, scale = se)[1]
    # calculates the upper and lower bounds using scipy

    for upper, mean, lower, y in zip(upper, mean, lower, cat):
        plt.plot((lower, mean, upper), (y, y, y), 'b.-')
        # for 'b.-': 'b' means 'blue', '.' means dot, '-' means solid line
    plt.yticks(
        range(len(n)),
        list(data.groupby(col1, as_index = False)[col2].count()[col1])
        )
```

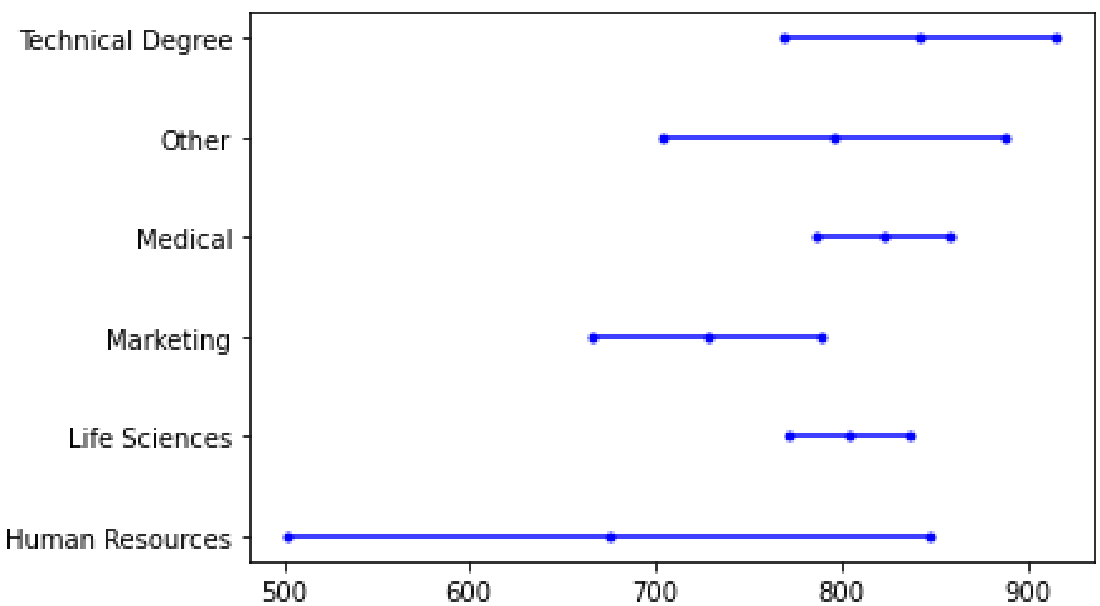
[38]:

```python
plot_diff_in_means(data = df,alpha=.95, col1 = 'Gender', col2 = 'DailyRate')
```



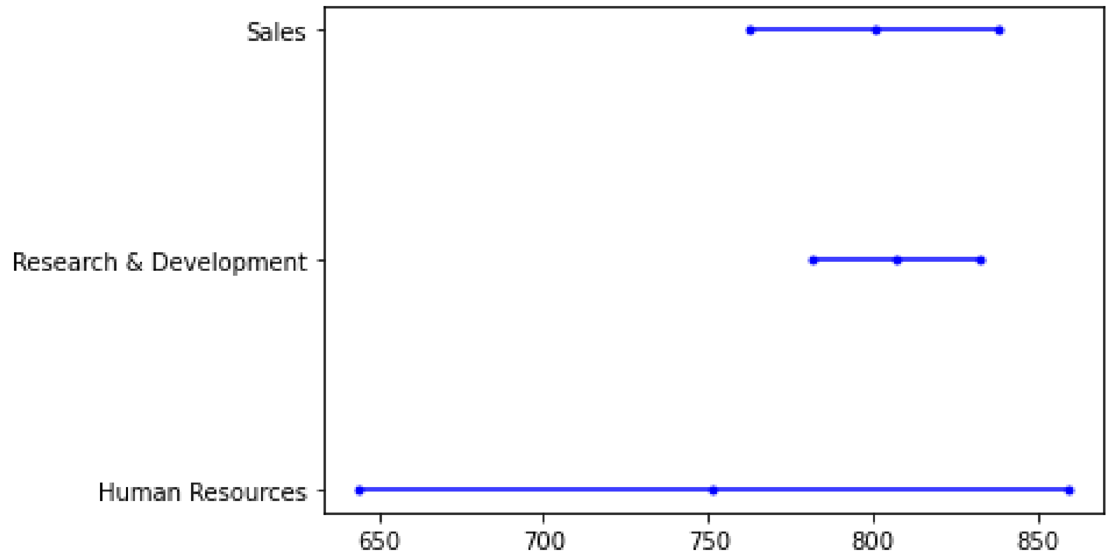

DailyRate and EducationField

[39]:
```
plot_diff_in_means(data = df,alpha=.95, col1 = 'EducationField', col2 =␣
↪'DailyRate')
```




DailyRate and Department

[40]:
```
plot_diff_in_means(data = df,alpha=.95, col1 = 'Department', col2 = 'DailyRate')
```




## 3 The proportion

Assume that we would like to estimate the probability $p$ to win an election for candidate A. We randomly select $n$ and consider $X$ the number of people reported that will vote for A.

The probability or the proportion $p$ is then estimated by

$$\widehat{p} = \frac{X}{n}$$

In most of the cases the number $n$, the sample size, is large and the probability distribution of $\widehat{p}$ is approximated with a Normal distribution with mean $\mu = p$ and variable $\sigma^2 = \dfrac{p(1-p)}{n}$. We can provide then, for a given $\alpha \in (0,1)$, a **Confidence interval** with level $1-\alpha$:

$$\mathrm{CI}_{1-\alpha}(p) = \left( \widehat{p} - z_{1-\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}},\ \widehat{p} + z_{1-\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} \right)$$

where $z_{1-\alpha/2}$ is the $z-$score associate to $1-\alpha/2$. It means, if $Z$ is a standard normal distribution, then

$$F_Z(z_{1-\alpha/2}) = 1 - \alpha/2$$

Where $F_Z$ is the CDF of $Z$.

**Example:** A Survey was conducted to estimate the probability $p$ to vote a candidate $A$. Among the 1200 participated in the Survey, 560 reported that will vote for the candidate A. Find an estimation of $p$ and its 95%-Confidence Interval.

[41]:
```
import statsmodels.api as sm
```

Importing the Function for computing proportion confidence intervals

[42]:
```
from statsmodels.stats.proportion import proportion_confint
```

[43]:
```
proportion_confint(count=560,     # Number of "successes"
                   nobs=1200,     # Number of trials
                   alpha=(1 - 0.95),
                   method='normal')   # when we use asymptotic normal␣
 ↪approximation
# Alpha, which is 1 minus the confidence level
```

[43]:
```
(0.4384399591955059, 0.49489337413782747)
```

There's also four oothers methods to compute the proportion confidence interval:

- agresti_coull : Agresti-Coull interval
- beta : Clopper-Pearson interval based on Beta distribution
- wilson : Wilson Score interval
- jeffreys : Jeffreys Bayesian Interval
- binom_test : experimental, inversion of binom_test

**Example:** We would like to compare the proportion of frequently traveling between the three departments. We start by computing first the contingency table between the variables `BusinessTravel` and `Departments`.

[44]:
```
import pandas as pd
```

[45]:
```
tab = pd.crosstab(df['BusinessTravel'], df['Department'],margins=True)
tab
```

[45]:

| Department | Human Resources | Research & Development | Sales | All |
|---|---|---|---|---|
| BusinessTravel | | | | |
| Non-Travel | 6 | 97 | 47 | 150 |
| Travel_Frequently | 11 | 182 | 84 | 277 |
| Travel_Rarely | 46 | 682 | 315 | 1043 |
| All | 63 | 961 | 446 | 1470 |

[46]:
```
table = sm.stats.Table(tab)
table.table
```

[46]:
```
array([[   6.,   97.,   47.,  150.],
       [  11.,  182.,   84.,  277.],
       [  46.,  682.,  315., 1043.],
       [  63.,  961.,  446., 1470.]])
```

[47]:
```
CI_HR=proportion_confint(count=table.table[1,0],nobs=table.table[3,0],alpha=(1-.
 ↪95))
CI_HR
```

[47]: (0.08086094446182195, 0.2683454047445272)

[48]:
```
CI_RD=proportion_confint(count=table.table[1,1],nobs=table.table[3,1],alpha=(1-.
↪95))
CI_RD
```

[48]: (0.16461369335918247, 0.214158419023752)

[49]:
```
CI_SL=proportion_confint(count=table.table[1,2],nobs=table.table[3,2],alpha=(1-.
↪95))
CI_SL
```

[49]: (0.1520547541182747, 0.22462686023150105)

[50]:
```
ci_travel = {}
ci_travel['Department'] = ['HR','RD','Sales']
ci_travel['lb'] = [CI_HR[0],CI_RD[0],CI_SL[0]]
ci_travel['ub'] = [CI_HR[1],CI_RD[1],CI_SL[1]]
df_ci= pd.DataFrame(ci_travel)
df_ci
```

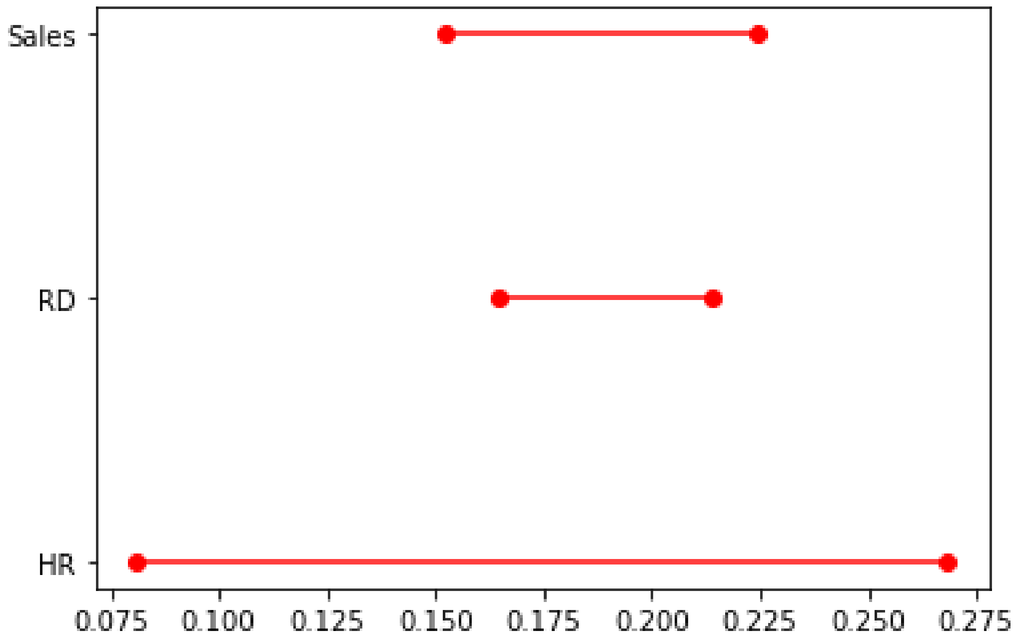
[50]:

| | Department | lb | ub |
|---|---|---|---|
| 0 | HR | 0.080861 | 0.268345 |
| 1 | RD | 0.164614 | 0.214158 |
| 2 | Sales | 0.152055 | 0.224627 |

[51]:
```
import matplotlib.pyplot as plt
```

[52]:
```
for lb,ub,y in zip(df_ci['lb'],df_ci['ub'],range(len(df_ci))):
    plt.plot((lb,ub),(y,y),'ro-')
plt.yticks(range(len(df_ci)),list(df_ci['Department']))
```

[52]:
```
([<matplotlib.axis.YTick at 0x1523b764e80>,
  <matplotlib.axis.YTick at 0x1523b764820>,
  <matplotlib.axis.YTick at 0x1523b75f670>],
 [Text(0, 0, 'HR'), Text(0, 1, 'RD'), Text(0, 2, 'Sales')])
```




## 4 The difference of two proportions

We observe now two independents samples with different sizes $n_1$ and $n_2$. We estimate from each sample a proportion. We aim to provide a **confidence interval** of the difference between these proportions. It can be expressed as follows:

$$\mathrm{CI}_{1-\alpha}(p_1 - p_2) = \left(\widehat{p}_1 - \widehat{p}_2 - z_{1-\alpha/2}\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}},\ \widehat{p}_1 - \widehat{p}_2 + z_{1-\alpha/2}\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}}\right)$$

We will write the following Python function

[53]:
```
import pandas as pd
import numpy as np
import scipy.stats as stats

def two_proprotions_confint(success_a, size_a, success_b, size_b, significance =␣
 ↪0.05):
    """
    A/B test for two proportions;
    given a success a trial size of group A and B compute
    its confidence interval;
    resulting confidence interval matches R's prop.test function
```

```
    Parameters
    ----------
    success_a, success_b : int
        Number of successes in each group

    size_a, size_b : int
        Size, or number of observations in each group

    significance : float, default 0.05
        Often denoted as alpha. Governs the chance of a false positive.
        A significance level of 0.05 means that there is a 5% chance of
        a false positive. In other words, our confidence level is
        1 - 0.05 = 0.95

    Returns
    -------
    prop_diff : float
        Difference between the two proportion

    confint : 1d ndarray
        Confidence interval of the two proportion test
    """
    prop_a = success_a / size_a
    prop_b = success_b / size_b
    var = prop_a * (1 - prop_a) / size_a + prop_b * (1 - prop_b) / size_b
    se = np.sqrt(var)

    # z critical value
    confidence = 1 - significance
    z = stats.norm(loc = 0, scale = 1).ppf(confidence + significance / 2)

    # standard formula for the confidence interval
    # point-estimtate +- z * standard-error
    prop_diff = prop_b - prop_a
    confint = prop_diff + np.array([-1, 1]) * z * se
    return prop_diff, confint
```

The Confidence interval of the difference between the proportions of frequently traveling between R&D and HR departments

[54]:
```
two_proprotions_confint(success_a=table.table[1,0],size_a=table.
↪table[3,0],success_b=table.table[1,1],size_b=table.table[3,1])
```

[54]:
```
(0.014782881588292635, array([-0.08217729,  0.11174306]))
```

The Confidence interval of the difference between the proportions of frequently traveling between R&D and Sales departments

[55]:
```
two_proprotions_confint(success_a=table.table[1,2],size_a=table.
↪table[3,2],success_b=table.table[1,1],size_b=table.table[3,1])
```

[55]:
```
(0.001045249016579347, array([-0.04289047,  0.04498097]))
```